

Symmetries of higher-spin current interactions in four dimensions

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Abstract

Current interaction of massless fields in four dimensions is shown to break $\mathfrak{sp}(8)$ symmetry of free massless equations of all spins down to the conformal symmetry $\mathfrak{su}(2,2)$. This breaking is in agreement with the form of nonlinear higher-spin field equations.

1 Introduction

The name of Igor Viktorovitch Tyutin is well known worldwide in the first place in relation with the BRST formalism. Undoubtedly Tyutin is one of the leaders of Russian science, having made outstanding contribution to relativistic quantum field theory far beyond the BRST formalism as such. Working with Igor Viktorovitch in the same group for a long time we had a great opportunity to fully appreciate both the true value of his scientific potential and the charm of his personality. For the volume in honor of Igor Tyutin's 75th birthday we are happy to contribute a paper where symmetries of relativistic systems are studied by the methods of unfolded dynamics having a much in common with the BRST approach [1].

As shown by Fronsdal [2], the infinite towers of free massless fields that appear in the $4d$ higher-spin (HS) gauge theory [3] exhibit $\mathfrak{sp}(8)$ symmetry which extends the conformal symmetry $\mathfrak{su}(2, 2)$. The latter acts on every spin $s = 0, 1/2, 1, 3/2, 2, \dots$. The generators from $\mathfrak{sp}(8)/\mathfrak{su}(2, 2)$ mix fields of different spins.

This observation suggests a manifestly $\mathfrak{sp}(8)$ -symmetric geometric realization of field equations of massless fields of all spins studied e.g. in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13] (and references therein). However attempts to extend this formalism to HS interactions were not successful [12]. The full nonlinear system of HS equations [14] possesses manifest $\mathfrak{sp}(4)$ symmetry rather than $\mathfrak{sp}(8)$. It was not clear however whether this is an artifact of the formalism or the $\mathfrak{sp}(8)$ symmetry is inevitably broken by interactions.

In this paper we show that HS current interactions necessarily break $\mathfrak{sp}(8)$ down to its conformal subalgebra $\mathfrak{su}(2, 2)$. Our analysis is based on the approach of [15] where it was shown that current interactions can be understood as a deformation of the two independent linear systems for fields of rank-one and rank-two, associated, respectively, with massless fields and conserved conformal currents in four space-time dimensions. Though each of these systems is $\mathfrak{sp}(8)$ symmetric it is not guaranteed that $\mathfrak{sp}(8)$ is preserved by the deformation responsible for interactions. Let us emphasise that our results do not rule out a possibility of introducing other interactions, the existence of which is indicated by the results of [16, 13], where nontrivial $\mathfrak{sp}(8)$ invariant three-point correlators were constructed.

The analysis of [15] was manifestly invariant under the AdS_4 symmetry algebra $\mathfrak{sp}(4)$. An important question not considered in [15] is to which extent the deformed system respects $\mathfrak{sp}(8)$. As shown below, in the interacting system, $\mathfrak{sp}(4)$ can be extended to the $4d$ conformal algebra $\mathfrak{su}(2, 2)$ but not to the full $\mathfrak{sp}(8)$. This implies that the current HS interactions break $\mathfrak{sp}(8)$ down to its conformal subalgebra $\mathfrak{su}(2, 2)$. Specifically, $\mathfrak{sp}(8)$ contains the helicity operator \mathcal{H} such that its centralizer in $\mathfrak{sp}(8)$ is $\mathfrak{u}(2, 2)$ spanned by the generators of conformal algebra $\mathfrak{su}(2, 2)$ and \mathcal{H} itself. As we show, the $\mathfrak{u}(1)$ symmetry generated by \mathcal{H} cannot be extended to the mixed system. Hence, the same is true for $\mathfrak{sp}(8)/\mathfrak{su}(2, 2)$.

We analyze the system within the unfolded form of [15] in the sector of gauge invariant curvature 0-forms which is free of subtleties due to gauge symmetries. One of the main results of this paper is that, rather unexpectedly, this setup turns out to be manifestly conformal invariant. The analysis in terms of 0-forms is more general than that in the sector of gauge fields where the conformal invariance is broken for massless fields of spins $s > 1$. The latter breaking can be resolved however on the AdS background [10].

The symmetry $\mathfrak{u}(1) \in \mathfrak{sp}(8)$ generated by the helicity operator \mathcal{H} describes electric-magnetic (EM) duality. Recall that the EM duality generalized to spins $s \geq 1$ is generated by \mathcal{H} , with respect to which (anti)self-dual solutions possess eigenvalues $(-)^s$. In the case of $s = 1$ this is the conventional EM duality. Since current interactions are known to break EM duality our conclusions are not too surprising. The breaking of $\mathfrak{sp}(8)$ considered in this paper occurs in the sector of 0-forms and cannot be restored via transition to a curved background in spirit of [10].

Our results are in agreement with the form of nonlinear HS equations [14] which also breaks EM duality. In fact, the form of nonlinear HS equations suggests that this breaking is of spontaneous type *a la* embedding tensor formalism in supergravity [17, 18].

In the rest of the paper we first recall relevant facts of the unfolded dynamics formalism in Section 2. In Section 3 we reformulate the problem of current interactions in terms of module deformation. The conformally invariant setup is worked out in Section 4. Symmetries of HS current interactions are verified in Section 5. Obtained results are briefly discussed in Section 6.

2 Unfolded dynamics

2.1 Unfolded equations

Unfolded dynamics formalism is most useful to control symmetries in a system. The idea of this approach was suggested and applied to the interacting HS gauge theory in [19, 20]. For more detail see also [3, 21, 22].

Let M^d be a d -dimensional manifold with coordinates x^n ($n = 0, 1, \dots, d-1$). Unfolded formulation of a linear or nonlinear system of differential equations and/or constraints in M^d assumes its reformulation in the first-order form

$$dW^\Phi(x) = G^\Phi(W(x)), \quad (2.1)$$

where $d = dx^n \frac{\partial}{\partial x^n}$ is the exterior derivative on M^d , $W^\Phi(x)$ is a set of degree- p_Φ differential forms and $G^\Phi(W)$ is some degree $p_\Phi + 1$ function of W^Φ

$$G^\Phi(W) = \sum_{n=1}^{\infty} f^\Phi_{\Omega_1 \dots \Omega_n} W^{\Omega_1} \wedge \dots \wedge W^{\Omega_n},$$

where $f^\Phi_{\Omega_1 \dots \Omega_n}$ are appropriately (anti)symmetrized structure coefficients and G^Φ satisfies the generalized Jacobi condition

$$G^\Omega(W) \wedge \frac{\partial G^\Phi(W)}{\partial W^\Omega} = 0. \quad (2.2)$$

Strictly speaking, formal consistency demands (2.2) be satisfied at $p_\Phi < d$ for a d -dimensional manifold where any $d+1$ -form is zero. Any solution of (2.2) defines a free differential algebra [23, 24]. The unfolded system is *universal* [21] if the generalized Jacobi identity is true independently of the dimension d , *i.e.*, W^Φ can be treated as local coordinates of some target superspace. Unfolded HS equations including those considered in this paper are universal.

For universal systems, equation (2.1) is invariant under the gauge transformation

$$\delta W^\Phi(x) = d\varepsilon^\Phi(x) + \varepsilon^\Omega(x) \wedge \frac{\partial G^\Phi(W(x))}{\partial W^\Omega(x)}, \quad (2.3)$$

where the gauge parameter $\varepsilon^\Phi(x)$ is a $(p_\Phi - 1)$ -form (which is zero for 0-forms $W^\Phi(x)$).

Local degrees of freedom are contained in the 0-form sector. This is a consequence of the Poincaré lemma since exact forms can be removed order by order by gauge transformation (2.3). It should be noted that on the language of unfolded equations local evolution of field is determined by values of 0-forms at some fixed point of space-time. The transition to a standard Cauchy problem is related to the decomposition of the fields into dynamical and auxiliary. The latter are expressed via derivatives of dynamical fields. In addition, unfolded equations impose differential equations on dynamical fields like Klein-Gordon, Dirac and other relativistic equations. Analysis of the dynamical content of unfolded equations is performed with the help of the σ_- -cohomology technics of [25] (see also [26]).

Possible degrees of freedom of closed p -forms which can be present due to nontrivial topology are not regarded as local.

A universal unfolded system can be uplifted to a larger space via the extension

$$d = dx^a \frac{\partial}{\partial x^a} \longrightarrow \hat{d} = d\hat{x}^A \frac{\partial}{\partial \hat{x}^A} = dx^a \frac{\partial}{\partial x^a} + dz^\alpha \frac{\partial}{\partial z^\alpha}, \quad (2.4)$$

where z^α are some additional coordinates. An extension of a universal unfolded system in the space M with coordinates x^a to a larger space \mathcal{M} with coordinates \hat{x}^A remains formally consistent. Since the restriction of the resulting system in \mathcal{M} to the original system in M is achieved by restricting \hat{d} to d , the local dynamical contents of the two systems are equivalent. Indeed, initial data are still given by values of 0-forms at any point of \mathcal{M} which can be chosen to be in M . The additional equations in \mathcal{M} reconstruct the dependence on z in terms of that in x . (Of course, this is true locally and the situation can change if z^α obey nontrivial boundary conditions.) Allowing simple extensions to larger spaces, universal unfolded systems provide an efficient tool used in particular for the extension of the formulation of massless fields from Minkowski space to the Lagrangian Grassmannian in [6] where the $\mathfrak{sp}(8)$ symmetry of HS multiplets acts geometrically.

2.2 Vacuum

The simplest class of universal free differential algebras is in one-to-one correspondence with Lie algebras. Indeed, let w^α be a set of 1-forms. If no other forms are involved (e.g., all of them are consistently set to zero in a larger system) the most general expression for $G^\alpha(w)$, that has to be a 2-form, is $G^\alpha(w) = -\frac{1}{2}f_{\beta\gamma}^\alpha w^\beta \wedge w^\gamma$. Consistency condition (2.2) and unfolded equations (2.1) impose, respectively, the Jacobi identity on the structure coefficients $f_{\beta\gamma}^\alpha$ of a Lie algebra \mathfrak{g} and the flatness condition on w^α

$$dw^\alpha + \frac{1}{2}f_{\beta\gamma}^\alpha w^\beta \wedge w^\gamma = 0. \quad (2.5)$$

Transformation law (2.3) yields the usual gauge transformation of the connection w

$$\delta w^\alpha(x) = D\varepsilon^\alpha(x) := d\varepsilon^\alpha(x) + f_{\beta\gamma}^\alpha w^\beta(x)\varepsilon^\gamma(x). \quad (2.6)$$

A flat connection $w(x)$ is invariant under the global transformations with the covariantly constant parameters

$$D\varepsilon^\alpha(x) = 0. \quad (2.7)$$

This equation is consistent by virtue of (2.5). Therefore, locally, it reconstructs $\varepsilon^\alpha(x)$ in terms of its values $\varepsilon^\alpha(x_0)$ at any given point x_0 . $\varepsilon^\alpha(x_0)$ are the moduli of the global symmetry \mathfrak{g} that is now recognized as the stability algebra of a given flat connection $w(x)$.

This example explains how \mathfrak{g} -invariant vacuum fields appear in the unfolded formulation. Namely, vacuum is understood as a solution of a nonlinear system possessing one or another symmetry \mathfrak{g} , which is usually described by a flat connection valued in a representation of \mathfrak{g} . Typically, an unfolded system that contains 1-forms w^α associated with some Lie algebra \mathfrak{g} admits a flat connection w^α as its \mathfrak{g} -symmetric vacuum solution. In the perturbative analysis, the vacuum connection w^α is assumed to be a solution of some nonlinear system and to be of order zero. Such description of the background geometry is coordinate independent. A manifest form of $w^\alpha(x)$ is not needed unless one is interested in explicit solutions in the specific coordinate system.

2.3 Free fields and Chevalley-Eilenberg cohomology

Let us linearize unfolded equation (2.1) around some vacuum flat connection w setting

$$W^\Omega = w^\Omega + \omega^\Omega, \quad (2.8)$$

where w is a flat connection of a Lie algebra \mathfrak{g} , which solves (2.5), while ω^Ω are differential forms of various degrees that are treated as small perturbations and enter the equations linearly in the lowest order. Consider first the sector of forms $\omega^i(x)$ of a given degree p_i (e.g., 0-forms) within the set $\omega^\Omega(x)$. Then G^i is bilinear in w and ω , i.e., $G^i = -w^\alpha(T_\alpha)^i_j \wedge \omega^j$. In this case condition (2.2) implies that the matrices $(T_\alpha)^i_j$ form a representation T of \mathfrak{g} in a vector space V where $\omega^i(x)$ are valued. Corresponding equation (2.1) is the covariant constancy condition

$$D_w \omega^i = d\omega^i + w^\alpha(T_\alpha)^i_j \wedge \omega^j = 0 \quad (2.9)$$

with $D_w \equiv d + w$ being the covariant derivative in the \mathfrak{g} -module V .

Equations (2.5) and (2.9) are invariant under gauge transformations (2.3) with

$$\delta \omega^i(x) = -\varepsilon^\alpha(x)(T_\alpha)^i_j \omega^j(x). \quad (2.10)$$

Once the vacuum connection is fixed, system (2.9) is invariant under the global symmetry \mathfrak{g} with the parameters satisfying (2.7).

This simple analysis has useful consequences. First of all, unfolding of any \mathfrak{g} -invariant linear system of partial differential equations implies its reformulated in terms of \mathfrak{g} -modules. Unfolded equations of the form (2.9) can be integrated in the pure gauge form

$$\omega^i(x) = g^i_j(x, x_0)\omega^j(x_0), \quad g^i_j(x, x_0) = P \exp - \int_{x_0}^x w^\alpha T_\alpha^i_j. \quad (2.11)$$

If $\tilde{\mathfrak{g}}$ is a larger Lie algebra that acts in V , $\mathfrak{g} \subset \tilde{\mathfrak{g}} \subset \mathfrak{gl}(V)$, it is also a symmetry of (2.9) simply because any flat \mathfrak{g} -connection is the same time a flat $\tilde{\mathfrak{g}}$ -connection. As a result, the Lie algebra $\mathfrak{gl}(V)$ of commutators of endomorphisms of V is the maximal symmetry of (2.9).

Let $\omega^{\mathbf{a}}(x)$ and $\omega^{\mathbf{b}}(x)$ be forms of different fixed degrees $p_{\mathbf{a}}$ and $p_{\mathbf{b}}$, say, $p_{\mathbf{a}} - p_{\mathbf{b}} = k \geq 0$. In the linearized approximation, one can consider G^{Φ} polylinear in the vacuum field w^{α} but still linear in the dynamical fields ω

$$G^{\mathbf{a}}(w, \omega) = -f_{\alpha_1 \dots \alpha_{k+1}, \mathbf{b}}^{\mathbf{a}} w^{\alpha_1} \wedge \dots \wedge w^{\alpha_{k+1}} \omega^{\mathbf{b}}. \quad (2.12)$$

Let $\omega^{\mathbf{b}}$ be a 0-form. The equation for $\omega^{\mathbf{b}}$ is a covariant constancy condition (2.9). The consistency condition (2.2) applied to (2.12) then literally implies that $f_{\alpha_1 \dots \alpha_{k+1}, \mathbf{b}}^{\mathbf{a}} w^{\alpha_1} \wedge \dots \wedge w^{\alpha_{k+1}}$ is a Chevalley-Eilenberg cocycle of \mathfrak{g} with coefficients in $V_l \otimes V_r^*$ where V_l is the module where $G^{\mathbf{a}}$ is valued while V_r^* is the module dual to that of $\omega^{\mathbf{b}}$. Coboundaries are dynamically empty because, as is easy to see, they can be removed by a field redefinition. Thus, in the unfolded formulation, the Chevalley-Eilenberg cohomology classifies possible nontrivial mixings between fields realized as differential forms of different degrees [10]. Specifically, if both $\omega^{\mathbf{a}}$ and $\omega^{\mathbf{b}}$ are 0-forms, the Chevalley-Eilenberg cohomology (2.12) describes a nontrivial deformation of the direct sum of the modules associated with $\omega^{\mathbf{a}}$ and $\omega^{\mathbf{b}}$. This case is most relevant to the analysis of this paper where current interactions are described as a deformation of the direct sum of the modules associated with fields and currents.

In presence of deformation (2.12) the system remains invariant under the global symmetry \mathfrak{g} which is still the part of the gauge symmetry that leaves invariant the vacuum fields w^{α} . Its action on ω^{Φ} is deformed however by (2.12) according to (2.3).

3 Higher-rank fields, nonlinear system and holography

Consider some \mathfrak{g} -symmetric free field-theoretical system. Its unfolded formulation always contains a set of 0-forms (describing matter fields and gauge invariant combinations of derivatives of the gauge fields in the system) that form a \mathfrak{g} -module S . Being the space of all local degrees of freedom in the system, S is closely related to the Hilbert space H of single-particle states, that is a space of normalizable (positive-frequency) solutions of free equations in the corresponding free quantum theory (S is usually complex equivalent to H [6]). Then the analogue of the space of two-particle states is $Sym S \otimes S$ and of the space M of all multiparticle states is

$$M(S) = \oplus_{r=0}^{\infty} S^r, \quad S^r = Sym \underbrace{S \otimes \dots \otimes S}_r. \quad (3.1)$$

where D is a vacuum covariant derivative in the \mathfrak{g} -module S . Let 0-forms field $C^i(x)$ of some unfolded system satisfy the vacuum covariant constancy condition

$$DC^i(x) = 0, \quad (3.2)$$

where D is the vacuum covariant derivative in \mathfrak{g} -module S . Let $C^i(x)$ be called rank-one field. Then a rank- r field $C^{i_1 \dots i_r}(x)$ is defined to be valued in S^r and satisfy the equation

$$D_r C^{i_1 \dots i_r}(x) = 0, \quad (3.3)$$

where D_r is the vacuum covariant derivative of \mathfrak{g} in S^r . According to Section 2.3, the maximal symmetry of the rank- r equation is $\mathfrak{gl}(S^r)$.

A basis of S^r is formed by *pure* fields being products of the rank-one fields

$$A_{i_1, \dots, i_r} C^{i_1}(x) \dots C^{i_r}(x). \quad (3.4)$$

(Note that, unlikely to the bilocal approach of [27, 28], the product is taken at the same x but for different values of the indices i_k which label a basis of S .) The space S^r is invariant under $\mathfrak{gl}(S)$ principally embedded into $\mathfrak{gl}(S^r)$, *i.e.*, acting on every field in (3.4).

The unfolded equations for the gauge invariant field strengths of massless fields of all spins in $4d$ Minkowski space are [20]

$$DC(y, \bar{y}|x) := dx^{\alpha\beta'} \left(\frac{\partial}{\partial x^{\alpha\beta'}} + \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\beta'}} \right) C(y, \bar{y}|x) = 0. \quad (3.5)$$

Here y^α and $\bar{y}^{\beta'}$ are auxiliary mutually conjugated commuting coordinates carrying two-component spinor indices $\alpha, \beta = 1, 2$; $\alpha', \beta' = 1, 2$ while $x^{\alpha\beta'}$ are Minkowski coordinates in two-component spinor notations.

The rank- r generalization of (3.5) is

$$D_r C^{(r)}(y, \bar{y}|x) := dx^{\alpha\beta'} \left(\frac{\partial}{\partial x^{\alpha\beta'}} + \frac{\partial^2}{\partial y_i^\alpha \partial \bar{y}_j^{\beta'}} \eta_{ij} \right) C^{(r)}(y, \bar{y}|x) = 0, \quad (3.6)$$

where $C^{(r)}(y, \bar{y}|x) = C(y_1, \dots, y_r, \bar{y}_1, \dots, \bar{y}_r|x)$ and η_{ij} is some nondegenerate $r \times r$ matrix. According to Section 2, Eq. (3.6) is $\mathfrak{sp}(8)$ symmetric simply because $\mathfrak{sp}(8)$ is generated by the operators

$$\left\{ Y_j^A, \frac{\partial}{\partial Y_j^B} \right\}, \quad \frac{\partial^2}{\partial Y_i^A \partial Y_j^B} \eta_{ij}, \quad Y_i^A Y_j^B \eta^{ij} \quad \left(Y_j^A = (y_j^\alpha, \bar{y}_j^{\beta'}) \right) \quad (3.7)$$

which act on the functions $C^{(r)}(y, \bar{y})$.

As shown in [26], the unfolded form of the current conservation condition is just the rank-two system while usual currents result from bilinear substitution (3.4). The construction of currents in the CC form in $4d$ Minkowski space was discussed in detail in [29]. The full current deformation of the HS field equations involves gauge 1-forms ω . The *r.h.s.s* of their field equations contain conserved currents analogously to the stress tensor on the *r.h.s.s* of the Einstein equations. It is this sector that contains most of usual current interactions. The rank-one 0-forms C describe gauge invariant combinations of space-time derivatives of ω . Current deformation of the equations on the gauge fields ω induces the deformation of the equations on C . Though this part of the deformation is less familiar, it is in a certain sense simpler being free of complications due to gauge symmetries. Hence, in this paper we focus on the sector of 0-forms.

In the noninteracting case the modules C and J for fields and currents obey independent unfolded field equations

$$DC = 0, \quad D_2 J = 0 \quad (3.8)$$

with covariant derivatives D (3.5) and D_2 (3.6). Both of these systems are $\mathfrak{sp}(8)$ invariant hence being consistent for arbitrary flat $\mathfrak{sp}(8)$ connection in $D = d + w$. Schematically, the interacting system has the form

$$DC = F(w, J), \quad D_2 J = 0, \quad (3.9)$$

where $F(w, J)$ describes interactions once J is realized in terms of bilinears of C . In [15], current interactions of $4d$ massless fields were described as a $\mathfrak{sp}(4)$ -invariant linear system that mixes rank-one massless fields C with rank-two current fields J . Group-theoretically, the unfolded system of [15] describes a deformation of the direct sum of $\mathfrak{sp}(4)$ -modules S of massless fields and S^2 of conserved currents to an indecomposable $\mathfrak{sp}(4)$ -module $S^{1,2}$ associated with the exact sequence¹

$$0 \longrightarrow S^2 \longrightarrow S^{1,2} \longrightarrow S \longrightarrow 0. \quad (3.10)$$

Here S^2 is a submodule of $S^{1,2}$ and $S = S^{1,2}/S^2$.

In [15] it was checked that the proper current deformation of the free field equations describes a $\mathfrak{sp}(4)$ -module where the action of the algebra $\mathfrak{sp}(4) \subset \mathfrak{sp}(8)$ is associated with the AdS_4 symmetry. However, the existence of the $\mathfrak{sp}(4)$ invariant deformation of (3.8) to (3.9) does not imply that the deformed action of $\mathfrak{sp}(4)$ can be extended to $\mathfrak{sp}(8)$, *i.e.*, it is not guaranteed that Eq. (3.9) can be consistently formulated for an arbitrary flat $\mathfrak{sp}(8)$ connection w . It can happen that deformation (3.9) only makes sense for connections ω valued in some subalgebra \mathfrak{h} of the symmetry algebra \mathfrak{g} of system (3.8), forming an \mathfrak{h} -module but not a \mathfrak{g} -module. In this paper we will show that the deformation remains consistent for the $\mathfrak{su}(2, 2)$ extension of $\mathfrak{sp}(4)$ but not beyond, *i.e.*, the interactions preserve conformal symmetry but not full $\mathfrak{sp}(8)$.

A few comments are now in order.

The proposed construction not only properly describes the field equations of massless fields in presence of currents but also gives a useful tool for reconstruction of Greens functions (at least in the gauge invariant sector of 0-forms). Indeed, in the 0-form sector that mixes massless 0-forms C with current 0-forms J , the deformed field equation has the covariant constancy form (2.9) for some representation of the Minkowski or AdS_4 symmetry. Hence, in the 0-form sector, the deformed equations can be solved in the pure gauge form (2.11). The representation is indecomposable, having triangular form since J contributes to the equation for C but not otherwise. The operator g^i_j (2.11) also has a triangular form, containing the off-diagonal term that maps J to C but not the other way around. Hence, given J , C is reconstructed in the form

$$C = G(J) + C_0, \quad (3.11)$$

where C_0 is an arbitrary solution of the undeformed field equations for C . Clearly, G is Greens function that reconstructs solutions of massless equations via currents on their *r.h.s.* (More precisely, this is gauge invariant Greens function that reconstructs gauge invariant field strengths of massless fields.) It would be interesting to apply this method for a practical computation.

The current deformation of free field equations derived from the perturbative expansion of the nonlinear HS equations of [20, 14] (see however [30, 31] on possible subtleties of such a

¹ In [10], the fields C^i were interpreted as coefficients in $C = C^i t_i$ in a module with basis elements t_i . As usual for dual modules, for this convention the arrows have to be reversed.

derivation) has more general form than (3.9) containing also the HS gauge connections $\omega(y, \bar{y}|x)$

$$DC + \omega \star C - C \star \tilde{\omega} = F(w, C), \quad D_2 J = 0, \quad (3.12)$$

$$D^{ad} \omega + \omega \star \omega = G(w, C), \quad (3.13)$$

where bilinear combinations of C should be identified with the current J , \star is the Weyl star product acting on functions of y and \bar{y} , D^{ad} is the adjoint covariant derivative, and $\tilde{\omega}(y, \bar{y}|x) := \omega(-y, \bar{y}|x)$ (for more detail see [20, 3]). A simple but important fact, which follows from the analysis of [20], is that the ω -dependent terms on the *l.h.s.* and the terms on the *r.h.s.* of Eq. (3.12) contribute to different sectors of the equations. Namely, let s be the spin of the field C in the first term of (3.12) while s_1 and s_2 be spins of the field constituents of $J \sim CC$. Then for $s \neq 0$ the F -terms are non-zero at

$$s \geq s_1 + s_2. \quad (3.14)$$

In other words, the gauge invariant currents J built from the 0-forms C have spin s obeying (3.14). The ω -dependent terms in (3.12) with ω and C carrying spins s_1 and s_2 contribute to the equation for the spin- s field C with $s < s_1 + s_2$. Note that this conclusion is in agreement with the results of [32] where the currents with spins beyond the region (3.14) were built in terms of the gauge connections ω . In Appendix we show that the ω -dependent terms in (3.12) do not contribute to the consistency check of Eq. (3.12) in the sector (3.14).

In this paper we focus on the sector of fields obeying (3.14), discarding the ω -dependent terms. Note that, beyond the lowest-order deformation, all kinds of terms are anticipated to be mixed nontrivially, *i.e.*, the contribution of the gauge connection ω cannot be discarded in any sector at the full nonlinear level.

The ω -independent part of deformation (3.9) admits an interesting interpretation from the AdS_4/CFT_3 duality perspective. As shown in [33], the boundary limit of the $4d$ 0-forms $C(y, \bar{y}|x)$ gives $3d$ conformal currents \mathcal{J} which are $3d$ rank-two fields. As a result, the holographic dual of the $4d$ current J bilinear in C is $\mathcal{J}\mathcal{J}$. Hence the holographic version of equation (3.9) has the form

$$\mathcal{D}\mathcal{J} = F(w, J(\mathcal{J})), \quad (3.15)$$

where \mathcal{D} is the $\mathfrak{sp}(4)$ covariant derivative at the boundary. For $F = 0$ this gives the unfolded current conservation condition at the boundary [33]. For $F \neq 0$, the current conservation condition is deformed. In [34] such a deformation was associated with “slightly broken HS symmetry”.

It should be stressed however that the deformed HS equations still respect global HS symmetries. The latter are neither broken nor even deformed, *i.e.*, the algebra of HS transformations remains unchanged. What is deformed is the transformation law. Indeed, the nonlinear HS equations can be linearized with respect to an arbitrary HS connection $w(Y|x)$ obeying the flatness condition $dw + w \star w = 0$. Since the full nonlinear HS equations are formally consistent, their perturbative expansion around w is also consistent at any order. Application of (2.3) to the vacuum field w gives the transformation law for global HS symmetries in a chosen background. (w can be set equal to its AdS_4 value upon differentiation in (2.3).) Preservation

of the global HS symmetries at the nonlinear level is not too surprising being analogous to the fact that a perturbative expansion of any general coordinate invariant theory around Minkowski background preserves Poincaré symmetry.

So far our consideration was a consequence of the previous work of [20, 15]. The new result of this paper is that the manifest symmetry of equation (3.15) is larger than the boundary conformal symmetry $\mathfrak{sp}(4)$. Surprisingly it gets enhanced to the $4d$ conformal symmetry $\mathfrak{su}(2, 2)$. Note however that this is a symmetry of equation (3.15) but not necessarily of the operator algebra of boundary currents.

4 Background $\mathfrak{u}(2, 2)$ connection

As sketched in Section 2, a system is \mathfrak{g} -symmetric if its unfolded equations are formally consistent for any flat connection w of \mathfrak{g} .

In terms of two-component spinors, the $\mathfrak{su}(2, 2) \sim \mathfrak{o}(4, 2)$ connections are $h^{\alpha\alpha'}$, ω_α^β , $\bar{\omega}_{\alpha'}^{\beta'}$, b and $f_{\alpha\alpha'}$. Extending $\mathfrak{su}(2, 2)$ to $\mathfrak{u}(2, 2)$ by a central helicity generator with the gauge connection \tilde{b} , the $\mathfrak{u}(2, 2)$ flatness conditions read

$$\begin{aligned} R^{\alpha\beta'} &:= dh^{\alpha\beta'} - \omega_\gamma^\alpha \wedge h^{\gamma\beta'} - \bar{\omega}_{\gamma'}^{\beta'} \wedge h^{\alpha\gamma'} = 0, \\ R_{\alpha\beta'} &:= df_{\alpha\beta'} + \omega_\alpha^\gamma \wedge f_{\gamma\beta'} + \bar{\omega}_{\beta'}^{\gamma'} \wedge f_{\alpha\gamma'} = 0, \\ R_\alpha^\beta &:= d\omega_\alpha^\beta + \omega_\alpha^\gamma \wedge \omega_\gamma^\beta - f_{\alpha\gamma'} \wedge h^{\gamma'\beta} = 0, \\ \bar{R}_{\alpha'}^{\beta'} &:= d\bar{\omega}_{\alpha'}^{\beta'} + \bar{\omega}_{\alpha'}^{\gamma'} \wedge \bar{\omega}_{\gamma'}^{\beta'} - f_{\gamma\alpha'} \wedge h^{\gamma\beta'} = 0. \end{aligned} \quad (4.1)$$

Lorentz connection is described by the traceless parts $\omega_\alpha^{L\beta}$ and $\bar{\omega}_{\alpha'}^{L\beta'}$ of ω_α^β and $\bar{\omega}_{\alpha'}^{\beta'}$, respectively, while their traces are associated with the gauge fields

$$b = \frac{1}{2}(\omega_\alpha^\alpha + \bar{\omega}_{\alpha'}^{\alpha'}), \quad \tilde{b} = \frac{1}{2}(\omega_\alpha^\alpha - \bar{\omega}_{\alpha'}^{\alpha'}). \quad (4.2)$$

Note that since b and \tilde{b} are one-component

$$b \wedge b = \tilde{b} \wedge \tilde{b} = 0. \quad (4.3)$$

The $\mathfrak{u}(2, 2)$ invariant rank- r unfolded equations are

$$D_{r_u}^{tw} C^{(r)}(y, \bar{y}|x) = 0, \quad (4.4)$$

$$D_{r_u}^{tw} := d - \omega^{L\alpha\beta} y_{j\alpha} \partial_\beta^j - \bar{\omega}^{L\alpha'\beta'} \bar{y}_{j\alpha'} \bar{\partial}_{\beta'}^j + f_{\alpha\alpha'} y_i^\alpha \bar{y}_j^{\alpha'} \eta^{ij} + h^{\alpha\alpha'} \partial_\alpha^i \bar{\partial}_{\alpha'}^j \eta_{ij} + b \mathcal{D}_r + \tilde{b} \mathcal{H}_r, \quad (4.5)$$

where

$$\mathcal{H}_r = \frac{1}{2}(y_j^\alpha \partial_\alpha^j - \bar{y}_j^{\alpha'} \bar{\partial}_{\alpha'}^j) \quad (4.6)$$

is a *rank -r helicity operator*,

$$\mathcal{D}_r = r + \frac{1}{2}(y_j^\alpha \partial_\alpha^j + \bar{y}_j^{\alpha'} \bar{\partial}_{\alpha'}^j) \quad (4.7)$$

is a *rank -r dilatation operator* and

$$\eta_{ij}\eta^{kj} = \delta_i^k, \quad \partial_\beta^j = \frac{\partial}{\partial y_j^\beta}, \quad \bar{\partial}_{\alpha'}^j = \frac{\partial}{\partial \bar{y}_j^{\alpha'}}, \quad i, j, k = 1, \dots, r.$$

The rank-one equation $D_{1\mathbf{u}}^{tw}C(y, \bar{y}|x) = 0$ describes the $\mathfrak{u}(2, 2)$ invariant $4d$ massless field equations in terms of the generalized Weyl tensors $C(y, \bar{y}|x)$.

For general rank, the conformal dimension Δ_r of a field component equals to the eigenvalue of \mathcal{D}_r . For a spin- s primary field of rank- r the conformal dimension is

$$\Delta_r = r + s. \quad (4.8)$$

The conformal covariant derivative $D_{r\mathbf{su}}^{tw}$ coincides with $D_{r\mathbf{u}}^{tw}$ (4.5) at $\tilde{b} = 0$

$$D_{r\mathbf{su}}^{tw} = D_{r\mathbf{u}}^{tw} \Big|_{\tilde{b}=0}. \quad (4.9)$$

Eigenvalues of \mathcal{H}_r (4.6) describe helicities of field components. At $r = 1$, $\mathcal{H} = \mathcal{H}_1$ is the usual helicity operator. Conformal algebra $\mathfrak{su}(2, 2)$ is the subalgebra of $\mathfrak{sp}(8)$ spanned by elements commuting with $\mathcal{H} \in \mathfrak{sp}(8)$ (4.6). More precisely, the centralizer of \mathcal{H} in $\mathfrak{sp}(8)$ is $\mathfrak{su}(2, 2) \oplus \mathfrak{u}(1)$ where $\mathfrak{u}(1)$ is generated by \mathcal{H} . Conformal algebra $\mathfrak{su}(2, 2)$ and \mathcal{H} act on states of definite helicities while $\mathfrak{sp}(8)$ mixes different helicities.

The AdS_4 geometry is described by the Lorentz connections $\omega^{L\alpha\beta}$, $\bar{\omega}^{L\alpha'\beta'}$ and vierbein $e^{\alpha\alpha'}$ of $\mathfrak{sp}(4, \mathbb{R}) \subset \mathfrak{su}(2, 2) \subset \mathfrak{sp}(8, \mathbb{R})$ via the substitution

$$h^{\alpha\alpha'} = \lambda e^{\alpha\alpha'}, \quad f_{\alpha\alpha'} = \lambda e_{\alpha\alpha'}, \quad b = \tilde{b} = 0, \quad (4.10)$$

which gives

$$D_{rads}^{tw} := d - \omega^{L\alpha\beta} y_{j\alpha} \partial_\beta^j - \bar{\omega}^{L\alpha'\beta'} \bar{y}_{j\alpha'} \bar{\partial}_{\beta'}^j + \lambda e_{\alpha\alpha'} y_i^\alpha \bar{y}_j^{\alpha'} \eta^{ij} + \lambda e^{\alpha\alpha'} \partial_\alpha^i \bar{\partial}_{\alpha'}^j \eta_{ij}. \quad (4.11)$$

Two-component indices are raised and lowered by the symplectic forms $\varepsilon_{\alpha\beta}$ and $\varepsilon_{\alpha'\beta'}$

$$A_\beta = A^\alpha \varepsilon_{\alpha\beta}, \quad A_{\beta'} = A^{\alpha'} \varepsilon_{\alpha'\beta'}, \quad A^\alpha = A_\beta \varepsilon^{\alpha\beta}, \quad A^{\alpha'} = A_{\beta'} \varepsilon^{\alpha'\beta'}. \quad (4.12)$$

5 Symmetries of current interactions

5.1 Structure of the current deformation

As shown in [15], in the unfolded dynamics approach, interactions of $4d$ massless fields of all spins with currents result from the linear problem describing a mixing of rank-one (massless particles) and rank-two (current) systems.

To make contact with [15], we set $\eta_{ij} = \delta_{ij}$, $\eta^{ij} = \delta^{ij}$ and introduce the new variables

$$\sqrt{2}y^{\pm\alpha} = y_1^\alpha \pm iy_2^\alpha, \quad \sqrt{2}\bar{y}^{\pm\alpha'} = \bar{y}_1^{\alpha'} \pm i\bar{y}_2^{\alpha'}, \quad \partial_{\pm\alpha} = \frac{\partial}{\partial y^{\pm\alpha}}, \quad \bar{\partial}_{\pm\alpha'} = \frac{\partial}{\partial \bar{y}^{\pm\alpha'}}. \quad (5.1)$$

In these variables rank-two covariant derivative (4.5) is

$$\begin{aligned}
D_{2u}^{tw} &= d - \left(\omega^{\alpha\beta} y^+_{\alpha} \partial_{+\beta} + \bar{\omega}^{\alpha'\beta'} \bar{y}^+_{\alpha'} \bar{\partial}_{+\beta'} + \omega^{\alpha\beta} y^-_{\alpha} \partial_{-\beta} + \bar{\omega}^{\alpha'\beta'} \bar{y}^-_{\alpha'} \bar{\partial}_{-\beta'} \right) \\
&+ f_{\alpha\alpha'} (y^{+\alpha} \bar{y}^{-\alpha'} + y^{-\alpha} \bar{y}^{+\alpha'}) + h^{\alpha\alpha'} (\partial_{+\alpha} \bar{\partial}_{-\alpha'} + \partial_{-\alpha} \bar{\partial}_{+\alpha'}) \\
&+ \frac{1}{2} b (4 + y^{j\alpha} \partial_{j\alpha} + \bar{y}^{j\alpha'} \bar{\partial}_{j\alpha'}) + \frac{1}{2} \tilde{b} (y^{j\alpha} \partial_{j\alpha} - \bar{y}^{j\alpha'} \bar{\partial}_{j\alpha'}) ,
\end{aligned} \tag{5.2}$$

while operator D_{2AdS}^{tw} (4.11) results from D_{2u}^{tw} (5.2) via substitution (4.10). Conserved currents $\mathcal{J}(y^{\pm}, \bar{y}^{\pm}|x)$ satisfy the rank-two current equations [35]

$$D_{2AdS}^{tw} \mathcal{J}(y^{\pm}, \bar{y}^{\pm}|x) = 0. \tag{5.3}$$

Evidently, Eq. (5.3) decomposes into a set of subsystems

$$D_{2AdS}^{tw} \mathcal{J}_h(y^{\pm}, \bar{y}^{\pm}|x) = 0, \quad \mathcal{H}_2 \mathcal{J}_h(y^{\pm}, \bar{y}^{\pm}|x) = h \mathcal{J}_h(y^{\pm}, \bar{y}^{\pm}|x) \tag{5.4}$$

characterized by different eigenvalues of the rank-two helicity operator \mathcal{H}_2 (4.6).

Deformed system (3.9) in AdS_4 has the form [15]

$$DC(y, \bar{y}|x) + \left\{ e^{\alpha\alpha'} y_{\alpha} \mathcal{F}_{\alpha'} \mathcal{J}(y^{\pm}, \bar{y}^{\pm}|x) + e^{\alpha\alpha'} \bar{y}_{\alpha'} \bar{\mathcal{F}}_{\alpha} \mathcal{I}(y^{\pm}, \bar{y}^{\pm}|x) \right\} \Big|_{y^{\pm}=\bar{y}^{\pm}=0} = 0, \tag{5.5}$$

$$D_2 \mathcal{J}(y^{\pm}, \bar{y}^{\pm}|x) = 0, \quad D_2 \mathcal{I}(y^{\pm}, \bar{y}^{\pm}|x) = 0$$

with $D = D_{1AdS}^{tw}$ (4.11), $D_2 = D_{2AdS}^{tw}$ and

$$\mathcal{F}_{\alpha'} = \sum_h \mathcal{F}_{h\alpha'}, \quad \mathcal{F}_{h\alpha'} = (F_h^+ \bar{\partial}_{+\alpha'} - F_h^- \bar{\partial}_{-\alpha'}) , \tag{5.6}$$

$$\bar{\mathcal{F}}_{\alpha} = \sum_h \bar{\mathcal{F}}_{h\alpha}, \quad \bar{\mathcal{F}}_{h\alpha} = (\bar{F}_h^+ \partial_{+\alpha} - \bar{F}_h^- \partial_{-\alpha}) , \tag{5.7}$$

$$F_h^{\pm} = \frac{\partial}{\partial \mathcal{N}_{\pm}} \sum_{n=0}^{2h} a_{n,2h} (\mathcal{N}_+)^n (\mathcal{N}_-)^{2h-n} \sum_{k \geq 0} \frac{(\bar{\mathcal{N}}_+ \mathcal{N}_- + \bar{\mathcal{N}}_- \mathcal{N}_+)^k}{k!(k+2h+1)!}, \quad \mathcal{N}_{\pm} = y^{\alpha} \partial_{\pm\alpha}, \quad \bar{\mathcal{N}}_{\pm} = \bar{y}^{\alpha'} \bar{\partial}_{\pm\alpha'}. \tag{5.8}$$

Here \mathcal{J} and \mathcal{I} can be independent rank-two fields. Complex conjugated equations are analogous.

The coefficients $a_{n,2h}$ remain arbitrary. Their absolute values reflect the freedom in normalization of currents of different spins while phases can be understood as resulting from EM-like duality transformations for different spins. Different phases correspond to different models. The freedom in such a phase was originally observed in [20] in the analysis of HS interactions at the same order as in this paper. However, since the analysis of [20] respected HS symmetries, the latter expressed phases of different spins in terms of a single phase parameter $\eta = \exp i\varphi$ which survives in the full nonlinear HS theory [14].

Note that the gluing operators \mathcal{F} and $\bar{\mathcal{F}}$ in (5.5) are such that only components \mathcal{J}_h of \mathcal{J} carrying rank-two helicities $h \geq -1$ contribute. Analogously, only \mathcal{I}_h with $h \leq 1$ contribute to

(5.5). For instance, for rank-one fields of definite helicity $\mathcal{H}_1 C_h = h C_h$, Eq. (5.5) yields for $h \geq 0$

$$\begin{aligned} D_{AdS}^{tw} C_h(y, \bar{y}|x) + e^{\alpha\alpha'} y_\alpha \mathcal{F}_{h\alpha'} \mathcal{J}_{h-1} \Big|_{y^\pm = \bar{y}^\pm = 0} &= 0, \\ D_{AdS}^{tw} C_{-h}(y, \bar{y}|x) + e^{\alpha\alpha'} \bar{y}_{\alpha'} \bar{\mathcal{F}}_{h\alpha} \mathcal{I}_{1-h} \Big|_{y^\pm = \bar{y}^\pm = 0} &= 0, \end{aligned} \quad (5.9)$$

$$D_{AdS}^{tw} C_0(y, \bar{y}|x) + e^{\alpha\alpha'} y_\alpha \mathcal{F}_{0\alpha'} \mathcal{J}_{-1} \Big|_{y^\pm = \bar{y}^\pm = 0} + e^{\alpha\alpha'} \bar{y}_{\alpha'} \bar{\mathcal{F}}_{0\alpha} \mathcal{I}_1 \Big|_{y^\pm = \bar{y}^\pm = 0} = 0$$

with $\mathcal{F}_{s\alpha'}$ (5.6) and $\bar{\mathcal{F}}_{s\alpha}$ (5.7).

The following simple properties are used below along with their complex conjugates:

$$[A(\mathcal{N}, \bar{\mathcal{N}}), y^{j\mu}] = y^\mu \frac{\partial}{\partial \mathcal{N}_j} A(\mathcal{N}, \bar{\mathcal{N}}), \quad \left[\frac{\partial}{\partial y^\mu}, A(\mathcal{N}, \bar{\mathcal{N}}) \right] = \frac{\partial}{\partial \mathcal{N}_j} A(\mathcal{N}, \bar{\mathcal{N}}) \frac{\partial}{\partial y^{j\mu}}, \quad (5.10)$$

$$A(\mathcal{N}, \bar{\mathcal{N}}) y^k{}_\alpha B(y^\pm) \Big|_{y^\pm = \bar{y}^\pm = 0} = y_\alpha \frac{\partial}{\partial \mathcal{N}_k} A(\mathcal{N}, \bar{\mathcal{N}}) B(y^\pm) \Big|_{y^\pm = \bar{y}^\pm = 0} \quad \forall A, B.$$

As shown in [15], F_h^\pm (5.8) obey

$$\begin{aligned} \frac{\partial}{\partial \mathcal{N}_-} F_h^+ + \frac{\partial}{\partial \mathcal{N}_+} F_h^- &= 0, \quad \left(\frac{\partial^2}{\partial \mathcal{N}_+ \partial \bar{\mathcal{N}}_-} + \frac{\partial^2}{\partial \mathcal{N}_- \partial \bar{\mathcal{N}}_+} - 1 \right) F_h^\pm = 0, \\ \left\{ 2 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} \right\} \left(\frac{\partial F_h^+}{\partial \bar{\mathcal{N}}_-} + \frac{\partial F_h^-}{\partial \bar{\mathcal{N}}_+} \right) - \mathcal{N}_- F_h^- - \mathcal{N}_+ F_h^+ &= 0, \\ \left\{ 2 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} \right\} \frac{\partial F_h^\pm}{\partial \bar{\mathcal{N}}_\pm} - \mathcal{N}_\mp F_h^\pm &= 0. \end{aligned} \quad (5.11)$$

$\bar{F}_h^\pm = \overline{F_h^\pm}$ obey the conjugated relations.

5.2 Conformal invariance of the deformation

To show that the $\mathfrak{sp}(4)$ invariant mixed system of [15] is conformal consider deformed equation (5.5) with $D = D_{1_{\text{su}}}^{tw}$, $D_2 = D_{2_{\text{su}}}^{tw}$ (4.9), $\bar{\mathcal{F}}_\alpha$ (5.6) and $\mathcal{F}_{\alpha'}$ (5.7). For simplicity we set $\mathcal{I} = 0$. Consistency of these equations restricts the deformation by the conditions

$$\begin{aligned} & bh^{\gamma\gamma'} y_\gamma \mathcal{F}_{\gamma'} \mathcal{J}(y^\pm, \bar{y}^\pm|x) \Big|_{y^\pm = \bar{y}^\pm = 0} + bh^{\gamma\gamma'} \left\{ \left(1 + \frac{1}{2} \left(y^\alpha \partial_\alpha + \bar{y}^{\alpha'} \bar{\partial}_{\alpha'} \right) \right) y_\gamma \mathcal{F}_{\gamma'} \right. \\ & - y_\gamma \mathcal{F}_{\gamma'} \left(2 + \frac{1}{2} \left(y^{+\alpha} \partial_{+\alpha} + \bar{y}^{+\alpha'} \bar{\partial}_{+\alpha'} + y^{-\alpha} \partial_{-\alpha} + \bar{y}^{-\alpha'} \bar{\partial}_{-\alpha'} \right) \right) \Big\} \mathcal{J}(y^\pm, \bar{y}^\pm|x) \Big|_{y^\pm = \bar{y}^\pm = 0} \\ & + h^{\gamma\gamma'} h^{\alpha\beta'} \left\{ \partial_\gamma \bar{\partial}_{\gamma'} y_\alpha \mathcal{F}_{\beta'} - y_\alpha \mathcal{F}_{\beta'} \left(\partial_{-\gamma} \bar{\partial}_{+\gamma'} + \partial_{+\gamma} \bar{\partial}_{-\gamma'} \right) \right\} \mathcal{J}(y^\pm, \bar{y}^\pm|x) \Big|_{y^\pm = \bar{y}^\pm = 0} \\ & + f^{\gamma\gamma'} h^{\alpha\beta'} \left\{ y_\gamma \bar{y}_{\gamma'} y_\alpha \mathcal{F}_{\beta'} - y_\alpha \mathcal{F}_{\beta'} \left(\bar{y}^{+\gamma'} y_{-\gamma}^- + \bar{y}^{-\gamma'} y_{+\gamma}^+ \right) \right\} \mathcal{J}(y^\pm, \bar{y}^\pm|x) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0. \end{aligned} \quad (5.12)$$

Note that the first term in (5.12) results from $dh^{\gamma\gamma'}$ via flatness conditions (4.1), accounting for the conformal dimension of the frame field $h^{\gamma\gamma'}$.

Using (5.8), (5.10) along with the decomposition

$$h^{\alpha\alpha'} \wedge h^{\beta\beta'} = \frac{1}{2} \varepsilon^{\alpha\beta} \overline{H}^{\alpha'\beta'} + \frac{1}{2} \varepsilon^{\alpha'\beta'} H^{\alpha\beta}, \quad H^{\alpha\beta} = h^\alpha_{\alpha'} \wedge h^{\beta\alpha'}, \quad \overline{H}^{\alpha'\beta'} = h_\alpha^{\alpha'} \wedge h^{\alpha\beta'}, \quad (5.13)$$

the h^2 term in (5.12) demands

$$\begin{aligned} H^{\gamma'\beta'} \left(\bar{\partial}_{\gamma'} \left\{ 2 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} \right\} - \mathcal{N}_+ \bar{\partial}_{-\gamma'} - \mathcal{N}_- \bar{\partial}_{+\gamma'} \right) \mathcal{F}_{\beta'} &= 0, \\ H^{\mu\alpha} y_\alpha \left(\bar{\partial}_{\gamma'} \partial_\mu - (\partial_{-\mu} \bar{\partial}_{+\gamma'} + \partial_{+\mu} \bar{\partial}_{-\gamma'}) \right) \mathcal{F}^{\gamma'} &= 0. \end{aligned} \quad (5.14)$$

These conditions hold true by virtue of (5.11) along with (5.6). The fh term is also zero by virtue of (5.11) and (5.6)

$$2f^{\gamma\gamma'} h^{\alpha\beta'} y_\gamma \bar{y}_{\gamma'} y_\alpha \left\{ 1 - \left(\frac{\partial}{\partial \mathcal{N}_-} \frac{\partial}{\partial \mathcal{N}_+} + \frac{\partial}{\partial \mathcal{N}_+} \frac{\partial}{\partial \mathcal{N}_-} \right) \right\} \mathcal{F}_{\beta'} \mathcal{J} (y^\pm, \bar{y}^\pm | x) \Big|_{y^\pm = \bar{y}^\pm = 0} = 0. \quad (5.15)$$

The bh term vanishes by virtue of (5.10)

$$bh^{\gamma\gamma'} y_\gamma \left\{ 1 + \frac{1}{2} \left(3 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} + \overline{\mathcal{N}}_k \frac{\partial}{\partial \overline{\mathcal{N}}_k} \right) - \frac{1}{2} \left(5 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} + \overline{\mathcal{N}}_k \frac{\partial}{\partial \overline{\mathcal{N}}_k} \right) \right\} \mathcal{F}_{\gamma'} \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} = 0. \quad (5.16)$$

Here it is important that the first term accounting for the conformal dimension of $h^{\gamma\gamma'}$, precisely compensates the difference between the rank-one and rank-two vacuum conformal dimensions.

This proves consistency of the conformal deformation of Eq. (5.5) with $D = D_{1_{\text{su}}}^{tw}$ and $D_2 = D_{2_{\text{su}}}^{tw}$ (4.9) hence implying conformal invariance of this system.

5.3 Inconsistency of the $\mathfrak{u}(2, 2)$ extension

Now we are in a position to show that the current deformation is not $\mathfrak{u}(2, 2)$ invariant.

Consider Eq. (5.5) with $D = D_{1_{\text{u}}}^{tw}$, $D_2 = D_{2_{\text{u}}}^{tw}$ (4.5) and F^j (5.8). In addition to Eq. (5.12), its consistency demands by virtue of (5.10) that

$$\begin{aligned} & \tilde{b} h^{\alpha\beta'} y_\alpha \left\{ \frac{1}{2} \left(1 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} - \overline{\mathcal{N}}_k \frac{\partial}{\partial \overline{\mathcal{N}}_k} \right) - \frac{1}{2} \left(-1 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} - \overline{\mathcal{N}}_k \frac{\partial}{\partial \overline{\mathcal{N}}_k} \right) \right\} \mathcal{F}_{\beta'} \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} \\ & + \tilde{b} h^{\alpha\beta'} \bar{y}_{\beta'} \left\{ \frac{1}{2} \left(-1 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} - \overline{\mathcal{N}}_k \frac{\partial}{\partial \overline{\mathcal{N}}_k} \right) - \frac{1}{2} \left(1 + \mathcal{N}_k \frac{\partial}{\partial \mathcal{N}_k} - \overline{\mathcal{N}}_k \frac{\partial}{\partial \overline{\mathcal{N}}_k} \right) \right\} \overline{\mathcal{F}}_\alpha \mathcal{I} \Big|_{y^\pm = \bar{y}^\pm = 0} \end{aligned} \quad (5.17)$$

should be zero. Here all terms cancel except for the vacuum contributions which do not because the helicity operator counts the difference between powers of y and \bar{y} while $y_\alpha F^\pm \bar{\partial}_{\pm\beta'}$ is proportional to $y \frac{\partial}{\partial \bar{y}^\pm}$. (Note that helicity of the frame field is zero.) As a result, (5.17) takes the form

$$\tilde{b} h^{\alpha\beta'} \left(y_\alpha \mathcal{F}_{\beta'} \mathcal{J} - \bar{y}_{\beta'} \overline{\mathcal{F}}_\alpha \mathcal{I} \right) \Big|_{y^\pm = \bar{y}^\pm = 0}. \quad (5.18)$$

This is however nonzero. Indeed, decomposition (5.9) brings $\tilde{b}h$ term (5.18) to the form

$$\tilde{b}h^{\alpha\beta'} \left\{ \sum_{h>0} \left(y_\alpha \mathcal{F}_{h\beta'} \mathcal{J}_{h-1} - \bar{y}_{\beta'} \bar{\mathcal{F}}_{h\alpha} \mathcal{I}_{1-h} \right) + y_\alpha \mathcal{F}_{0\beta'} \mathcal{J}_{-1} - \bar{y}_{\beta'} \bar{\mathcal{F}}_{0\alpha} \mathcal{I}_1 \right\} \Big|_{y^\pm = \bar{y}^\pm = 0}. \quad (5.19)$$

Since the terms \mathcal{J}_{h-1} and \mathcal{I}_{1-h} are independent for $h > 0$, they should vanish separately which would imply that the current deformation is trivial. For $h = 0$ the two terms have different total (*i.e.*, rank-one) helicity and hence, again, should vanish separately.

An attempt to compensate the $\tilde{b}h$ -term in (5.17) by an additional \tilde{b} -dependent deformation term $\tilde{b}G\tilde{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0}$ for some operator G and rank-two field $\tilde{\mathcal{J}}$ fails because the modified consistency condition then would imply

$$\tilde{b}h^{\alpha\beta'} \left(y_\alpha \mathcal{F}_{\beta'} \mathcal{J} - \bar{y}_{\beta'} \bar{\mathcal{F}}_\alpha \mathcal{I} \right) \Big|_{y^\pm = \bar{y}^\pm = 0} = \tilde{b}D_u^{tw} G \tilde{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0} \equiv \tilde{b}D_{su}^{tw} G \tilde{\mathcal{J}} \Big|_{y^\pm = \bar{y}^\pm = 0}. \quad (5.20)$$

However if (5.20) admitted a solution, by $\tilde{b} \wedge \tilde{b} = 0$ (4.3) this would imply that the original $\mathfrak{su}(2, 2)$ current deformation $h^{\alpha\beta'} y_\alpha \mathcal{F}_{\beta'} \mathcal{J} \Big|_{y^\pm = \bar{y}^\pm = 0} + h^{\alpha\beta'} \bar{y}_{\beta'} \bar{\mathcal{F}}_\alpha \mathcal{I} \Big|_{y^\pm = \bar{y}^\pm = 0}$ is trivial which is not the case.

Thus it is shown that the current deformation of free field equations does not allow a $\mathfrak{u}(2, 2)$ -symmetric extension of the conformal deformation. Since the helicity operator \mathcal{H} (4.6) is a Cartan element of $\mathfrak{sp}(8)$, this implies that the $\mathfrak{sp}(8)$ invariant form of the current interactions is ruled out as well, *i.e.*, HS current interactions necessarily break $\mathfrak{sp}(8)$ down to $\mathfrak{su}(2, 2)$.

6 Discussion

The current deformation of the HS equations is shown to break the $\mathfrak{sp}(8)$ symmetry of the free massless field equations down to its conformal subalgebra. The analysis is performed in terms of the gauge invariant field strengths. To rule out the full $\mathfrak{sp}(8)$ symmetry it was enough to show the $\mathfrak{u}(1)$ helicity symmetry, which is a part of $\mathfrak{sp}(8)$, is inconsistent with the current interactions. This is not too surprising since the $\mathfrak{u}(1)$ helicity symmetry describes EM duality transformations known to be broken by the current interactions.

Our conclusions are in agreement with the structure of nonlinear HS field equations of [14] which contain a free phase parameter $\eta = \exp i\varphi$ forming a one-dimensional representation of the $\mathfrak{u}(1)$ helicity transformations that relates inequivalent HS theories associated with different values of φ . Hence the symmetry $\mathfrak{u}(1)$ of EM duality is not a symmetry of a HS theory with given η , mapping one theory to another. As such the parameter η is reminiscent of the embedding tensor introduced in [17, 18] to describe different models of supergravity. It would be also interesting to check its possible relation with the parameter of ω -deformation in supergravity in the context of ABJM theory [36] argued to play an important role in HS holography [37].

A useful viewpoint is to treat η as a VEV for some field affected by the symmetry $\mathfrak{u}(1)$ of EM duality. It would be interesting to look for a Higgs-like field in the nonlinear HS theory of [14] that would break $\mathfrak{sp}(8)$ down to the conformal algebra allowing to treat the $\mathfrak{sp}(8)$ symmetry

of the HS theory as spontaneously broken. Further extension of these ideas can lead to better insight into possible origin of duality symmetries in HS theory in spirit of the discussion of [38]

Note that being described in terms of a doubled set of oscillators $z_\alpha, y_\alpha, \bar{z}_{\dot{\alpha}}, \bar{y}_{\dot{\alpha}}$, the nonlinear equations of [14] do have a spontaneously broken $\mathfrak{sp}(8)$ symmetry. A possible relation between the two $\mathfrak{sp}(8)$ symmetries is not direct however because the one associated with the additional oscillators $z_\alpha, \bar{z}_{\dot{\alpha}}$ does not act properly on the free fields while that broken by the current interactions does.

Also let us stress that the proof of conformal invariance of the current interactions was given in the sector of 0-forms associated with the gauge invariant HS curvatures. We do not expect it to be extendable to the sector of gauge fields in the standard setup. (Recall that linearized Einstein equations are not conformal invariant in terms of the metric tensor while their consequences for the Weyl tensor are.) This can probably be achieved, however, in AdS_4 using the approach of [10].

It should be mentioned that the $\mathfrak{sp}(8)$ -invariant formalism extended at the level of free fields to HS 1-form connections in [10] contains a doubled set of all fields including the gauge fields. In this approach EM duality rotates the fields in the doublet. As is well known, for a doubled set of fields EM duality is much easier to achieve [39, 40, 41, 42, 43]. Hence one might expect that the obstruction of this paper can be avoided in the setup of [10]. Unfortunately we were not able to proceed along these lines. Basically the difficulty remains the same as in this paper if the current is constructed from the 0-form rank fields of the same type, *i.e.*, $J_{ii} \sim C_{ii}C_{ii}$ with $i = 0, 1$ in notations of [10], while other options do not have much sense at all from the perspective of matching of the left and right hand sides of the deformed equations.

Another interesting point is that, as shown in [40] (see also [44] and references therein), within the Hamiltonian-like approach breaking manifest Lorentz symmetry, EM duality can be realized as a manifest symmetry even at the Lagrangian level. The negative conclusion on the possibility to preserve manifest duality invariance was achieved in this paper in the manifestly Lorentz covariant approach, *i.e.*, it was shown that the maximal subalgebra of $\mathfrak{sp}(8)$ that contains the Lorentz subalgebra $\mathfrak{sl}(2|\mathbb{C})$ and admits a consistent current deformation is $\mathfrak{su}(2, 2)$ which does not contain the EM duality generator. The remaining question is whether there exists a subalgebra $\mathfrak{h} \in \mathfrak{sp}(8)$ that does not contain $\mathfrak{sl}(2|\mathbb{C})$ but contains the helicity generator \mathcal{H} and respects some current-like deformation.

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Appendix. Inequalities

The action of D on the ω -dependent terms in (3.12) gives by virtue of (3.13)

$$G(w, C) \star C - C \star \tilde{G}(w, C) . \quad (\text{A.1})$$

In the linearized approximation $G(w, C)$ is

$$G(w, C)(y, \bar{y}) = \eta \bar{H}^{\gamma' \beta'} \frac{\partial^2}{\partial \bar{y}^{\gamma'} \partial \bar{y}^{\beta'}} C(0, \bar{y}) + \bar{\eta} H^{\alpha \beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0), \quad (\text{A.2})$$

where η is a complex parameter. For given spins s_1 and s_2 of the first and second factors of C the η -dependent part of the first term of (A.1) is proportional to

$$X = \int d\bar{s} d\bar{t} \exp(i \bar{s}_{\alpha'} \bar{t}^{\alpha'}) \bar{H}^{\alpha' \beta'} \bar{t}_{\alpha'} \bar{t}_{\beta'} C_{s_1}(0, \bar{y} + \bar{s}) C_{s_2}(y, \bar{y} + \bar{t}). \quad (\text{A.3})$$

This gives

$$2s_1 = \bar{N}_{\bar{y}_1} + \bar{N}, \quad 2s_2 = |-N_y + \bar{N}_{\bar{y}_2} + \bar{N} - 2|, \quad 2s = |-N_y + \bar{N}_{\bar{y}_2} + \bar{N}_{\bar{y}_1}|, \quad (\text{A.4})$$

where s is a spin of X , N_y is the degree of y , $\bar{N}_{\bar{y}_1}$ and $\bar{N}_{\bar{y}_2}$ are the degrees of \bar{y} in the first and second factors of C , respectively, while \bar{N} is the degree of the integration parameter of \bar{s} equal to that of \bar{t} . Since $\bar{H}^{\alpha' \beta'} \bar{t}_{\alpha'} \bar{t}_{\beta'}$ has degree two in \bar{t}

$$\bar{N} \geq 2. \quad (\text{A.5})$$

The main fact is that X can be nonzero only if

$$s < s_1 + s_2. \quad (\text{A.6})$$

The proof is straightforward.

For

$$N_y \geq \bar{N}_{\bar{y}_2} + \bar{N} - 2, \quad N_y \leq \bar{N}_{\bar{y}_2} + \bar{N}_{\bar{y}_1} \quad (\text{A.7})$$

(A.4) yields

$$2s_1 = \bar{N}_{\bar{y}_1} + \bar{N}, \quad 2s_2 = N_y - \bar{N}_{\bar{y}_2} - \bar{N} + 2, \quad 2s = \bar{N}_{\bar{y}_1} + \bar{N}_{\bar{y}_2} - N_y. \quad (\text{A.8})$$

Assuming that $s \geq s_1 + s_2$ one has from (A.8)

$$\bar{N}_{\bar{y}_1} + \bar{N}_{\bar{y}_2} - N_y > \bar{N}_{\bar{y}_1} + \bar{N} + N_y - \bar{N}_{\bar{y}_2} - \bar{N}, \quad \bar{N}_{\bar{y}_2} \geq N_y + 1, \quad (\text{A.9})$$

that by virtue of (A.7) yields $\bar{N} \leq 1$, in contradiction with (A.5).

For

$$N_y \geq \bar{N}_{\bar{y}_2} + \bar{N} - 2, \quad N_y \geq \bar{N}_{\bar{y}_2} + \bar{N}_{\bar{y}_1} \quad (\text{A.10})$$

Eq. (A.4) yields

$$2s_1 = \bar{N}_{\bar{y}_1} + \bar{N}, \quad 2s_2 = N_y - \bar{N}_{\bar{y}_2} - \bar{N} + 2, \quad 2s = -\bar{N}_{\bar{y}_1} - \bar{N}_{\bar{y}_2} + N_y. \quad (\text{A.11})$$

Assuming that $s \geq s_1 + s_2$ (A.11) leads to the contradiction $-\bar{N}_{\bar{y}_1} \geq 1$.

For

$$N_y \leq \bar{N}_{\bar{y}_2} + \bar{N} - 2, \quad N_y \leq \bar{N}_{\bar{y}_2} + \bar{N}_{\bar{y}_1} \quad (\text{A.12})$$

Eq. (A.4) yields

$$2s_1 = \overline{N}_{\bar{y}_1} + \overline{N}, \quad 2s_2 = -N_y + \overline{N}_{\bar{y}_2} + \overline{N} - 2, \quad 2s = \overline{N}_{\bar{y}_1} + \overline{N}_{\bar{y}_2} - N_y. \quad (\text{A.13})$$

For $s \geq s_1 + s_2$ this yields $\overline{N} \leq 1$, in contradiction with (A.5).

For

$$N_y \leq \overline{N}_{\bar{y}_2} + \overline{N} - 2, \quad N_y \geq \overline{N}_{\bar{y}_2} + \overline{N}_{\bar{y}_1} \quad (\text{A.14})$$

(A.4) yields

$$2s_1 = \overline{N}_{\bar{y}_1} + \overline{N}, \quad 2s_2 = -N_y + \overline{N}_{\bar{y}_2} + \overline{N} - 2, \quad 2s = -\overline{N}_{\bar{y}_1} - \overline{N}_{\bar{y}_2} + N_y. \quad (\text{A.15})$$

At $s \geq s_1 + s_2$ this gives $N_y \geq \overline{N}_{\bar{y}_1} + \overline{N} + \overline{N}_{\bar{y}_2} - 1$ in contradiction with (A.14). This finishes the proof of inequality (A.6). The proof for the other terms in (A.1) is analogous.

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